All solutions to graded problems must be typed in LaTeX. All other problems can be typed or written, but must be completed. Graded problems must be organized in numerical order. All problem sets (both graded and non-graded) must be uploaded to Canvas as a single PDF document. Students can find the grading scale in our Syllabus.

Graded

Exercise 1. Let $X = \{a, b, c\}$. Which of the following collections are topologies on X. If a collection is not a topology, explain why it is not.

- $\{X, \{a\}, \{b\}, \{a, b\}\}$
- $\{X, \emptyset, \{a\}, \{b\}\}$
- $\{X, \emptyset, \{a\}\}$
- $\{X, \emptyset, \{a, b\}, \{b, c\}\}$
- $\{\{a\}, \{b\}, \{a, b\}\}$

Exercise 2. Consider the collection of subsets of the real line $\mathscr{L} = \{(x, \infty) : x \in \mathbb{R}\} \cup \{\mathbb{R}, \emptyset\}$. Prove that $\mathscr{L}$ is a topology on $\mathbb{R}$.

Non-Graded

Exercise 3. Under what conditions are the finite complement topology and the discrete topology the same topology for a fixed set?

Exercise 4. Let X be a nonempty set, and let $x \in X$. Define

$$\mathscr{T} = \{U \subseteq X : x \in U\} \cup \{\emptyset\}.$$

Prove that $\mathscr{T}$ is a topology on X.